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RHEODYNAMICS OF NONLINEARLY VISCOPLASTIC LIQUIDS

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Experimental results are given for the flow of an anomalous liquid and the possibility of a description of the flow curve that is invariant with respect to the geometry of the transporting medium is discussed.

There has recently been a sharp increase in the number of experimental and theoretical works devoted to the flow of non-Newtonian liquids. Of particular interest, because of their wide distribution, is the case of viscoplastic liquids, which are characterized by anomalous viscosity and also plastic properties such that, beyond some limiting shear stress, the liquid begins to flow.

The rheodynamic study of such systems is also of interest, since it affords the possibility of improving the technological processes in polymer reprocessing, in the production of composite materials and paint and varnish coatings, in petroleum reprocessing and elsewhere in the petroleum industry, and so on.

The extensive experimental material that has been accumulated on the flow of viscoplastic liquids in transporting media of various geometries [1, 2] indicates that the dependence of the tangential stress τ on the shear-rate gradient $\dot{\gamma}$ is nonlinear. For the solution of specific problems, the flow curves are approximated by one of the rheological models (Bingham—Shvedov, Balkley—Herschel, Caisson, etc.); note that the rheological parameters of the

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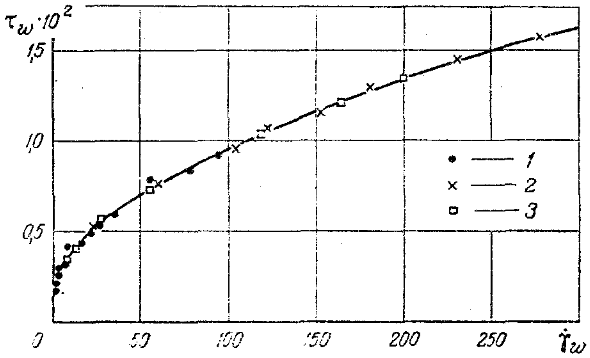


Fig. 1. Rheological curve for petroleum of the Kyurovdag deposit: 1) porous medium; 2) capillary viscometer; 3) slot model. τ_w, N/m²; $\dot{\gamma}_w$, sec⁻¹.

same liquid in the framework of the model adopted are different depending on the geometry of the transporting medium (circular capillary, porous medium, plane slot, etc.). This approach leads to practical inconvenience and severely restricts the possibility of complex investigations of many technological processes.

The present work discusses experimental results for the flow of non-Newtonian liquids in transporting media of various geometries (circular cylinder, porous medium, and plane slot) and demonstrates the invariance of the flow curve in the consistent variables proposed. The liquids taken in the experiments were Vapor (a lubricating oil for steam-engine cylinders), a mixture of 75% Vapor and 25% bright stock (a high-quality oil), and petroleum from a number of deposits in the Azneft' combine, at various temperatures (291-353°K). These liquids were chosen because, on the one hand, they are widely used in the study of processes of petroleum refining and petroleum extraction and, on the other hand, these media have distinctly nonlinear viscoplastic properties.

The rheometric results are analyzed in consistent variables, which are very convenient in practice; they are chosen on the basis of the Hagen—Poiseuille, Schiller, and Darcy formulas

$$\tau_w = \frac{\Delta P R_h}{l}, \tag{1}$$

$$\dot{\gamma}_w = \xi \frac{u_m}{R_h}. \tag{2}$$

It is possible to determine ξ using a method based on the analogy between laminar flow of a liquid and a large prism of the same cross section.

Taking into account that [3]

$$Q = \frac{\Delta P F^4}{16\pi^2 \mu I_0 l}, \tag{3}$$

we have

$$\frac{\Delta P R_h}{l} = \tau_w = \mu \frac{16\pi^2 I_0 R_h^2}{F^3} \frac{u_m}{R_h},$$

i.e.,

$$\xi = \frac{16\pi^2 I_0 R_h^2}{F^3}. \tag{4}$$

Note that this formula cannot be used in the case of an annular cross section.

In the course of the experimental investigation, the invariance of the flow curves in the coordinates $\tau_w - \dot{\gamma}_w$ with respect to the linear dimensions of the transporting medium was established. In addition, having plotted the rheological flow curves of the liquids in the various transporting media, it is possible to establish that all the rheometric data belong to a single curve; the data for the porous medium are mainly at moderate values of $\dot{\gamma}_w$ and those for a capillary and a slot are mainly at high values. As an example, Fig. 1 gives the

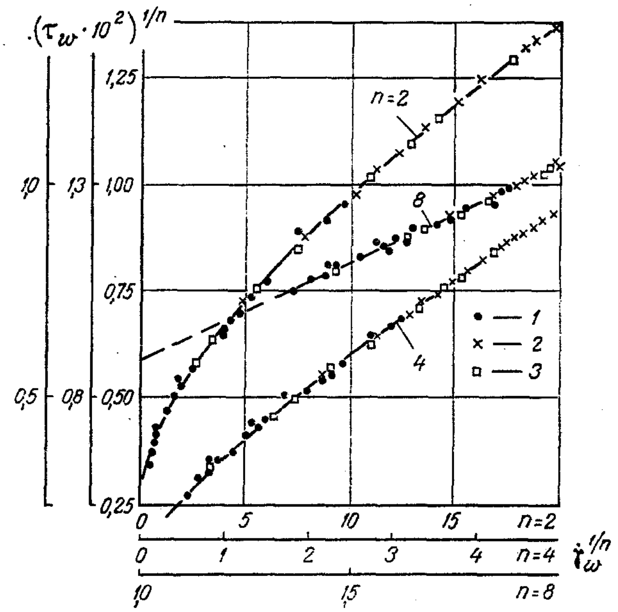


Fig. 2. Curves of $\tau_w^{1/n}$ versus $\dot{\gamma}_w^{1/n}$ for petroleum.

rheometric data for petroleum (at T = 291°K) in a capillary viscometer (d = 1.5, 2, 4 mm; l/d = 400), in a porous medium (κ = 16.52 darcy), and in a slot model (δ = 1.24 mm, b = 43 mm, l = 256 mm).

Rheological curves of this kind provide a true picture of the stress—strain state of the liquids and are independent of the geometry of the transporting medium. Note particularly that the unification of the flow curves is made possible by the use of the consistent variables proposed for transporting media of different geometries (see Table 1).

It is appropriate to note that an attempt of this kind to unify the flow curves of anomalous liquids was made earlier [2]. However, the rheological formula chosen for the porous medium was the Kozeni—Kármán formula, the parameters m and d of which are difficult to determine, and as a result the method was unsuitable for practical calculations. It should also be mentioned that, as noted in [4], the consistent variables proposed cannot be used to determine the velocity field and the local stresses nor to solve boundary-value problems of rheodynamics and convective heat and mass transfer.

If the flow curve is to be used in hydrodynamic investigations and calculations of technological processes, it is necessary to find an analytic representation of the dependence $\tau_w = f(\dot{\gamma}_w)$. Analysis of experimental results indicates that the generalized Caisson model, proposed in [4], gives the best fit for the flow curve over a fairly wide range of $\dot{\gamma}_w$.

Thus a rheological model of the form

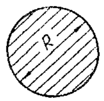
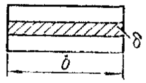
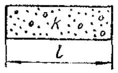
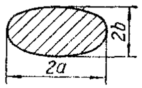
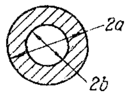
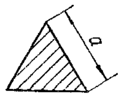
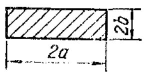
$$\tau^{1/n} = \tau_0^{1/n} + (\eta\dot{\gamma})^{1/n}, \tag{5}$$

where $n = 2^i$ (i = 1, 2, 3, ...), in the consistent variables proposed describes the flow of anomalous liquids in transporting media of various geometries.

The flow curves in Fig. 1 are plotted in the coordinates $\tau_w^{1/n}-\dot{\gamma}_w^{1/n}$ (n = 2, 4, 8) in Fig. 2. As is evident in Fig. 2, the curves become more rectilinear as n increases, and for n = 8 the curve is rectilinear over the whole range of $\dot{\gamma}_w$. The rheological parameters of the liquid were determined using mathematical statistics (τ_0 = 1.34 N/m², $\eta = 0.91 \cdot 10^{-3}$ N·sec/m²).

Note that further increase in n does not always lead to rectilinear rheological curves. It is found that there is a limiting value of n beyond which increase in n leads to distortion of the rheological curve in the opposite direction. As an illustration, data are given in Figs. 3 and 4 for capillary geometry (d = 2 mm, l/d = 400) for different petroleum of the same deposit at various temperatures; it is evident that, if the flow curve is rectilinear at n = 4, it is convex with respect to the $\dot{\gamma}_w$ axis at n = 8.

TABLE 1. Consistent Variables for Various Transporting Media

Transporting medium	Cross section	Flow rate of Newtonian liquid formula	Hydraulic radius R_h	Av. tang. stress over perim. τ_w	Shear-rate gradient $\dot{\gamma}_\omega$
Circular cylindrical tube		$Q = \frac{\pi \Delta P R^4}{8 \mu l}$	$\frac{R}{2}$	$\frac{\Delta P R}{2l}$	$2 \frac{u_m}{R_h}$
Plane tube (slot, crack)		$Q = \frac{\Delta P b \delta^3}{12 \mu l}$	$\frac{\delta}{2}$	$\frac{\Delta P \delta}{2l}$	$3 \frac{u_m}{R_h}$
Porous medium		$Q = \frac{\kappa F \Delta P}{\mu l}$	$\sqrt{\kappa}$	$\frac{\Delta P \sqrt{\kappa}}{l}$	$\frac{u_m}{R_h}$
Tube of elliptic cross section		$Q = \frac{\pi \Delta P a^3 b^3}{4 \mu l (a^2 + b^2)}$	$\frac{ab}{a+b}$	$\frac{\Delta P a b}{l(a+b)}$	$\frac{4(1+\alpha^2)}{(1+\alpha)^2} \frac{u_m}{R_h}$
Annular ring		$Q = \frac{\pi \Delta P a^4}{8 \mu l}\left[1 - \alpha^4 + \frac{(1-\alpha^2)^2}{\ln \alpha}\right]$	$\frac{1}{2}(a-b)$	$\frac{\Delta P (a-b)}{2l}$	$\frac{2(1-\alpha)^2}{1+\alpha^2+\frac{1-\alpha^2}{\ln \alpha}} \frac{u_m}{R_h}$
Tube of triangular cross section		$Q = \frac{\sqrt{3} \Delta P a^4}{320 \mu l}$	$\frac{\sqrt{3}}{12} a$	$\frac{\sqrt{3} \Delta P a}{12 l}$	$\frac{5}{3} \frac{u_m}{R_h}$
Tube of rectangular cross section		$Q = f\left(\frac{a}{b}\right) \frac{\Delta P a^2 b^2}{4 \mu l}$	$\frac{ab}{a+b}$	$\frac{\Delta P a b}{l(a+b)}$	$\frac{16\alpha}{(1+\alpha)^2 f(\alpha)} \frac{u_m}{R_h}$

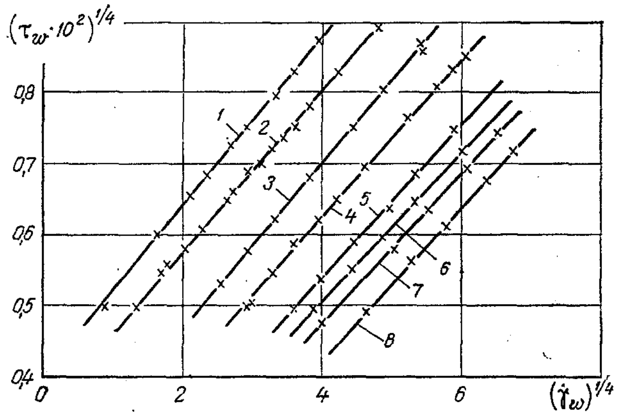


Fig. 3. Rheological curves for petroleum at different temperatures: 1) T = 291°K; 2) 294°K; 3) 297.5°K; 4) 301°K; 5) 304.5°K; 6) 307.5°K; 7) 313°K; 8) 323°K.

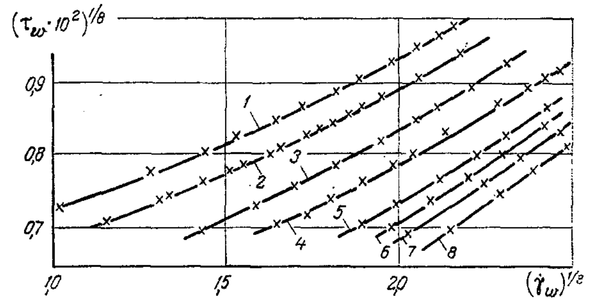


Fig. 4. Curve of $\tau_w^{1/8}$ versus $\dot{\gamma}_w^{1/8}$ for petroleum. Notation as in Fig. 3.

The data of numerous experiments show that n is determined by the shear-rate gradient: n = 2 for $\dot{\gamma}_w \geqslant 400$ sec^{-1}; n = 4 for $\dot{\gamma}_w \geqslant 16$ sec^{-1}; and n = 8 for $\dot{\gamma}_w \geqslant 0.0075$ sec^{-1}.

Thus, if information is available on the limits of variation of $\dot{\gamma}_w$ (i.e., if n is known), it is possible to pass from a three-parameter model to a two-parameter model.

NOTATION

τ, tangential stress; $\dot{\gamma}$, shear-rate gradient; τ_w, mean tangential stress over the perimeter; $\dot{\gamma}_w$, apparent shear-rate gradient; ΔP, pressure drop; $u_m = Q/F$, mean velocity; Q, volume flow rate of liquid; F, cross-sectional area; l, length of transporting medium; R_h, hydraulic radius; ξ, parameter characterizing the geometry of the transporting medium; μ, viscosity of Newtonian liquid; τ_0, yield point; η, structural viscosity; I_0, polar moment of inertia of cross section; κ, permeability of porous medium; δ, R, a, b, linear dimensions of various transporting media (see Table 1); $\alpha = b/a$.

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